Chapter 30

1

Sources of the magnetic field

1

Lecture 01

2

Biot-savart Law

2

Biot-Savart Law – Introduction

3

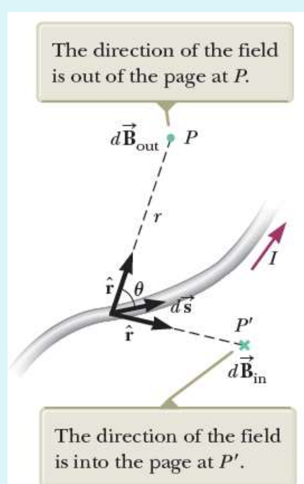
- Biot and Savart conducted experiments on the force exerted by an electric current on a nearby magnet.
- They arrived at a mathematical expression that gives the magnetic field at some point in space due to a current.
- The magnetic field described by the Biot-Savart Law is the field *due to* a given current carrying conductor.

3

Biot-Savart Law – Observations

4

- The vector $d\vec{B}$ is perpendicular to both $d\vec{s}$ and to the unit vector $\hat{r}$ directed from $d\vec{s}$ toward P.
- The magnitude of $d\vec{B}$ is inversely proportional to r^2, where r is the distance from $d\vec{s}$ to P.
- The magnitude of $d\vec{B}$ is proportional to the current and to the magnitude of the length element $d\vec{s}$.
- The magnitude of $d\vec{B}$ is proportional to $\sin\theta$, where θ is the angle between the vectors $d\vec{s}$ and $\hat{r}$.



4

Biot-Savart Law – Equation

5

- The observations are summarized in the mathematical equation called the **Biot-Savart law**:

$$d\vec{B} = \frac{\mu_\circ}{4\pi} \frac{I\, d\vec{s} \times \hat{r}}{r^2}$$

- To find the total field:

$$\vec{B} = \frac{\mu_\circ I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2}$$

- The constant $\mu_\circ$ is called the **permeability of free space.**

$$\mu_\circ = 4\pi \times 10^{-7}\ T.m/A$$

5

Thin, Straight Conductor - 1

Friday, 29 January, 2021 21:26

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.
R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

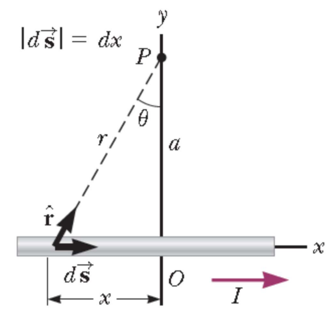
Consider a thin, straight wire of finite length carrying a constant current I and placed along the x axis as shown. Determine the magnitude and direction of the magnetic field at point P due to this current.

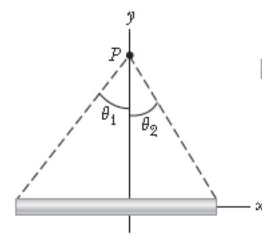
Answer:

$$B = \frac{\mu_o I}{4\pi a}(\sin\theta_1 + \sin\theta_2)$$

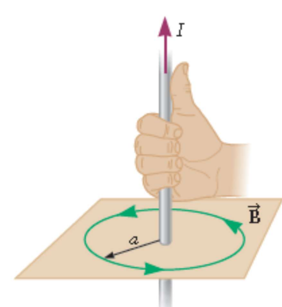
For an infinitely long, straight wire, where $\theta_1 \cong \theta_2 \cong \frac{\pi}{2}$

$$B = \frac{\mu_o I}{2\pi a}$$





- The magnetic field lines are circles concentric with the wire and lie in planes perpendicular to the wire.
- The magnitude of $\vec{B}$ is constant on any circle of radius a is:
$$B = \frac{\mu_o I}{2\pi a}$$
- The direction of $\vec{B}$: grasp the wire with the right hand, positioning the thumb along the direction of the current. The four fingers wrap in the direction of the magnetic field.
- The magnetic field line has no beginning and no end. Rather, it forms a closed loop.



OR: note the position of θ

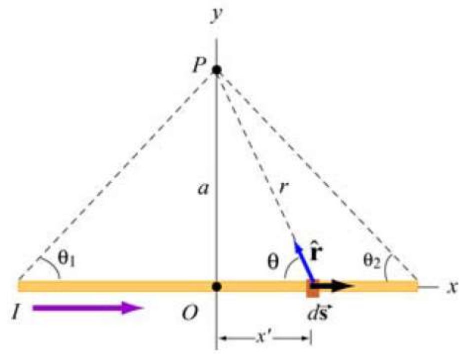
Answer:

$$B = \frac{\mu_o I}{4\pi a}(\cos\theta_1 + \cos\theta_2)$$

For an infinitely long, straight wire,

where $\theta_1 \cong \theta_2 \cong 0$

$$B = \frac{\mu_o I}{2\pi a}$$



magnetic Field of a single wire

Monday, 5 April, 2021 22:05

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.
- R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- ✓ H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A long, straight conductor carries a $1.0\ A$ current. At what distance from the axis of the conductor does the resulting magnetic field have magnitude $B = 0.5 \times 10^{-4}\ T$?

$$B = \frac{\mu_o I}{2\pi a} \rightarrow a = \frac{\mu_o I}{2\pi B}$$

$$a = \frac{4\pi \times 10^{-7} \times 1}{2\pi \times 0 \cdot 5 \times 10^{-4}} = 4 \times 10^{-3} m$$

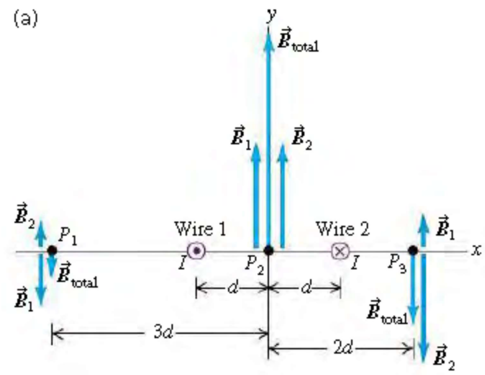
magnetic Field of two wires

Monday, 5 April, 2021 22:07

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

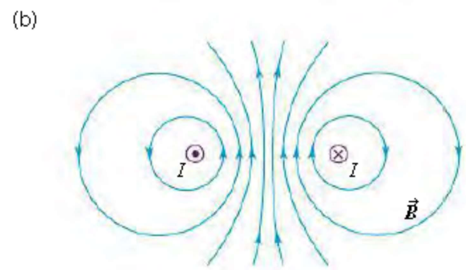
- R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

The figure shows an end-on view of two long, straight, parallel wires perpendicular to the xy-plane, each carrying a current I but in opposite directions. Find $\vec{B}$ at points P_1, P_2, and P_3.



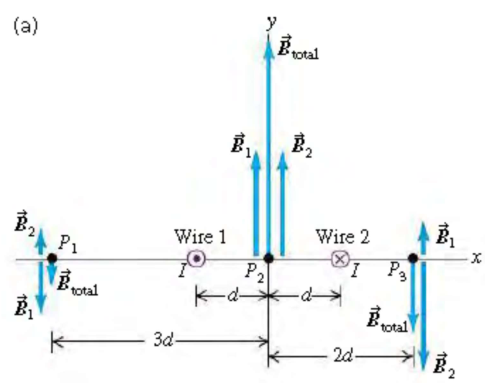
$$B = B_1 - B_2 = \frac{\mu_0 I}{2\pi(2d)} - \frac{\mu_o I}{2\pi(4d)} = \frac{\mu_0 I}{8\pi d}$$

$$\vec{B}_{P1} = \frac{\mu_0 I}{8\pi d}(-\hat{\jmath})$$



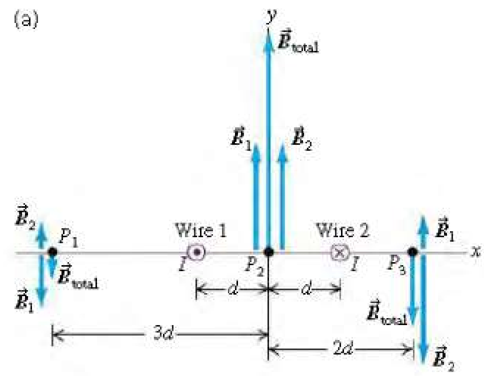
$$B = B_1 + B_2 = \frac{\mu_0 I}{2\pi(d)} + \frac{\mu_o I}{2\pi(d)} = \frac{\mu_0 I}{\pi d}$$

$$\vec{B}_{P2} = \frac{\mu_0 I}{\pi d}(\hat{\jmath})$$



$$B = B_2 - B_1 = \frac{\mu_0 I}{2\pi(d)} - \frac{\mu_o I}{2\pi(3d)} = \frac{\mu_0 I}{3\pi d}$$

$$\vec{B}_{\mathrm{P3}} = \frac{\mu_0 I}{3\pi d}(-\hat{j})$$



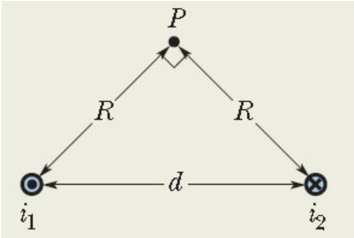
Thin, Straight Conductor - 2

Tuesday, 2 March, 2021 21:53

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- [ ] R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- [x] J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- [ ] H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- [ ] H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

The figure shows two long parallel wires carrying currents i_1 and i_2 in opposite directions. What are the magnitude and direction of the net magnetic field at point P? Assume the following values: $i_1 = 15\,A$, $i_2 = 32\,A$, and $d = 5.3\,cm$.



(1) The net magnetic field $\vec{B}$at point $P$is the vector sum of the magnetic fields due to the currents in the two wires.

(2) We can find the magnetic field due to any current by applying the Biot – Savart law to the current. For points near the current in a long straight wire, that law leads to Eq. 29-4.

Finding the vectors: In Fig. 29-8*a*, point P is distance R from both currents i_1 and i_2.Thus, Eq. 29-4 tells us that at point P those currents produce magnetic fields$\vec{B}_1$ and$\vec{B}_2$ with magnitudes

$$B_1 = \frac{\mu_0 i_1}{2\pi R}\ and\ B_2 = \frac{\mu_0 i_2}{2\pi R}.$$

In the right triangle of Fig. 29-8*a*, note that the base angles (between sides $R$and d) are both$45°$. This allows us to write cos $45° = R/d$and replace $R$with $d$cos 45°. Then the field magnitudes $B_1$and B_2become

$$B_1 = \frac{\mu_0 i_1}{2\pi d\ cos\ 45°}\ \text{ and }\ B_2 = \frac{\mu_0 i_2}{2\pi d\ cos\ 45°}.$$

To find the directions of $\vec{B}_1$and$\vec{B}_2$, we apply the right-hand rule of Fig. 29-4 to each current in Fig. 29-8*a*.

$\vec{B}_1$must be directed upward to the left as drawn in Fig. 29-8*b*.

$\vec{B}_2$is directed upward to the right as drawn in Fig. 29-8*b*.

Because$\vec{B}_1$ and$\vec{B}_2$ are perpendicular to each other, they form the legs of a right triangle, with $\vec{B}$as the hypotenuse. The Pythagorean theorem then gives us

$$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2\pi d(cos\,45°)}\sqrt{i_1^2 + i_2^2}$$

$$= \frac{(4\pi\times10^{-7}T\cdot m/A)\sqrt{(15A)^2+(32A)^2}}{(2\pi)(5.3\times10^{-2}m)(cos\ 45°)}$$

$$= 1.89\times10^{-4}T \approx 190\mu T.$$

The angle ϕ between the directions of$\vec{B}$ and$\vec{B}_2$ in Fig. 29-8*b* follows from

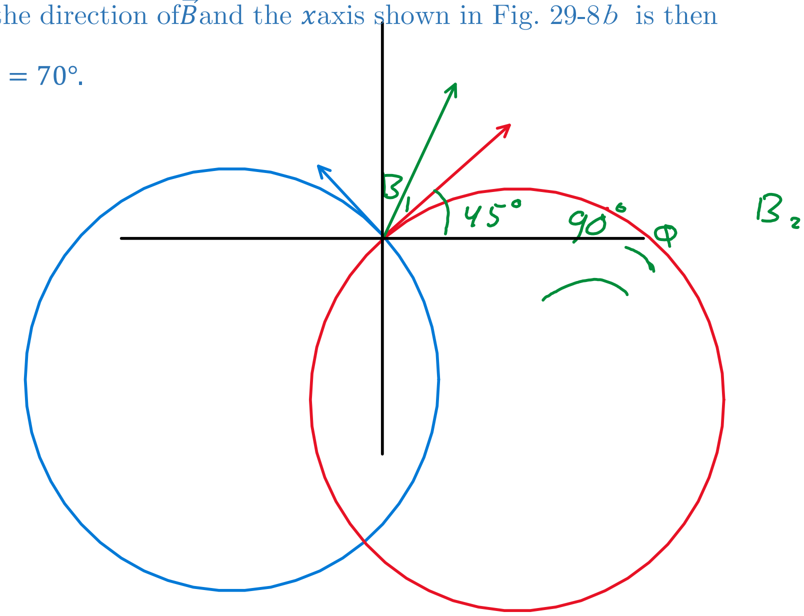
$$\phi = tan^{-1}\frac{B_1}{B_2},$$

which, with $B_1$and B_2as given above, yields

$$\phi = tan^{-1}\frac{i_1}{i_2} = tan^{-1}\frac{15A}{32A} = 25°.$$

The angle between the direction of $\vec{B}$ and the x axis shown in Fig. 29-8b is then

$$\phi + 45° = 25° + 45° = 70°.$$



A circular arc of current - 1

Thursday, 4 February, 2021 16:16

Calculate the magnetic field at point O for the current-carrying wire segment shown in the figure. The wire consists of two straight portions and a circular arc of radius a, which subtends an angle θ.

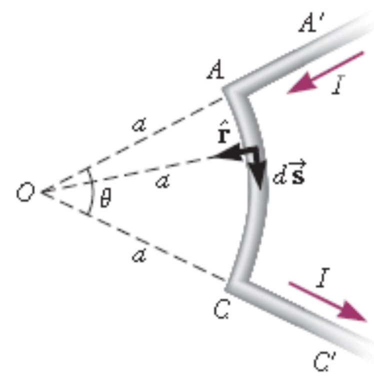
Answer:

$$B = \frac{\mu_o I}{4\pi a}\theta$$

θ: in RADIAN

At the center of a circular wire loop, where: $\theta = 2\pi$

$$B = \frac{\mu_o I}{2a}$$

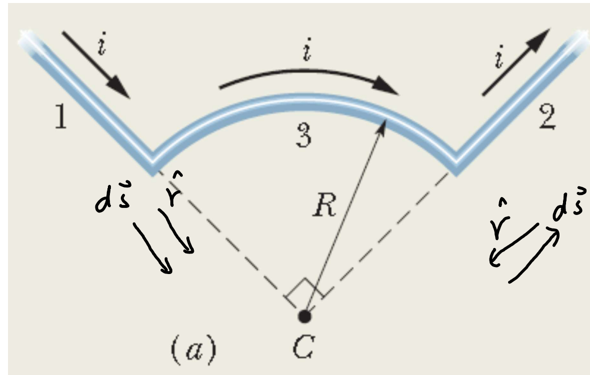


A circular arc of current - 2

Tuesday, 2 March, 2021 21:59

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

The wire shown in the figure carries a current i and consists of a circular arc of radius R and central angle $\frac{\pi}{2}\ rad$, and two straight sections whose extensions intersect the center C of the arc. What magnetic field (magnitude and direction) does the current produce at C?



Straight sections: For any current-length element in section 1, the angle θ between$d\vec{s}$ and $\hat{r}$ is zero (Fig. 29-7*b*); so Eq. 29-1 gives us

$$dB_1 = \frac{\mu_0}{4\pi}\frac{ids\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{ids\sin 0}{r^2} = 0.$$

Thus, the current along the entire length of straight section 1 contributes no magnetic field at C:

$$B_1 = 0.$$

The same situation prevails in straight section 2, where the angle θ between $d\vec{s}$and$\hat{r}$ for any current-length element is 180°.Thus,

$$B_2 = 0.$$

Circular arc: Application of the Biot–Savart law to evaluate the magnetic field at the center of a circular arc leads to Eq. 29-9 ($B = \mu_0 i\phi/4\pi R$). Here the central angle ϕ of the arc is $\pi/2$rad. Thus from Eq. 29-9, the magnitude of the magnetic field$\vec{B}_3$ at the arc's center $C$is

$$B_3 = \frac{\mu_0 i(\pi/2)}{4\pi R} = \frac{\mu_0 i}{8R}.$$

To find the direction of$\vec{B}_3$, we apply the right-hand rule displayed in Fig. 29-4. Mentally grasp the circular arc with your right hand as in Fig. 29-7*c*, with your thumb in the direction of the current.The direction in which your fingers curl around the wire indicates the direction of the magnetic field lines around the wire. They form circles around the wire, coming out of the page above the arc and going into the page inside the arc. In the region of point C (inside the arc), your fingertips point *into the plane* of the page. Thus,$\vec{B}_3$ is directed into that plane.

only the circular arc produces a magnetic field at point C. Thus, we can write the magnitude of the net field $\vec{B}$as

$$B = B_1 + B_2 + B_3 = 0 + 0 + \frac{\mu_0 i}{8R} = \frac{\mu_0 i}{8R}.$$

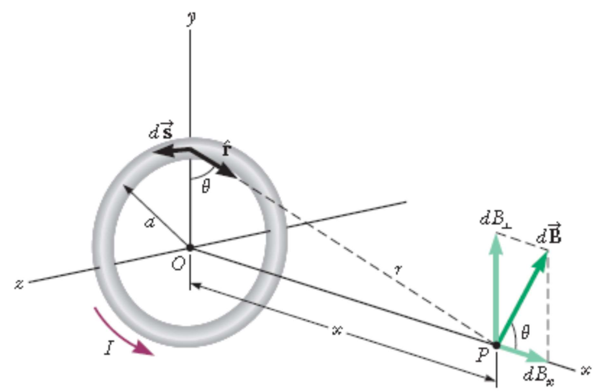
The direction of$\vec{B}$ is the direction of$\vec{B}_3$ —namely, into the plane of Fig. 29-7.

Circular Current Loop

Thursday, 4 February, 2021 16:44

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.
- R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

Consider a circular wire loop of radius a located in the yz plane and carrying a steady current I as shown. Calculate the magnetic field at an axial point P a distance x from the center of the loop.



Answer:

By symmetry:

$$B_\perp = 0$$

$$B_x = \frac{\mu_o I a^2}{2\,(a^2 + x^2)^{3/2}}$$

When $x \gg a$:

$$B_x \cong \frac{\mu_o I a^2}{2\,x^3}$$

When $x = 0$:

$$B_x = \frac{\mu_o I}{2\,a}$$

Magnetic field of a coil

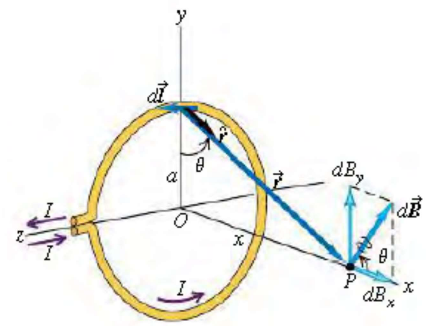
Tuesday, 6 April, 2021 08:30

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A coil consisting of 100 circular loops with radius $0.60\ m$ carries a $5.0\ A$ current.

- Find the magnetic field at a point along the axis of the coil, 0.80 m from the center.
- Along the axis, at what distance from the center of the coil is the field magnitude $1/8$ as great as it is at the center?



$$B = N\frac{\mu_0 I a^2}{2(a^2+x^2)^{3/2}} = \frac{100\times 4\pi\times 10^{-7}\times 5\times 0\cdot 6^2}{2(0.6^2+0.8^2)^{3/2}} = 113\times 10^{-6}T$$

$$B\Big|_x = \frac{1}{8}B\Big|_{x=0}$$

$$\frac{N\mu_0 I a^2}{2(a^2+x^2)^{3/2}} = \frac{1}{8}\frac{N\mu_0 I a^2}{2(a^2+0^2)^{3/2}}$$

$$\frac{8}{(0.6^2+x^2)^{3/2}} = \frac{1}{0.6^3}$$

$$\frac{2}{(0.6^2+x^2)^{1/2}} = \frac{1}{0.6} \rightarrow \frac{4}{0.6^2+x^2} = \frac{1}{0.36}$$

$$x^2 = 4\times 0.36 - 0.6^2 \Rightarrow x = \mp 1.04m$$

$$x = \mp 1.04m$$